Title:	System and Method for Performing a Progressive Second Price Auction
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#### AMENDMENTS TO THE SPECIFICATION:

# 1. Please amend the paragraph beginning on page 6, line 15 as follows:

Step 103 retrieves a list of bids previously made for the resource in a particular auction which have been previously sorted by decreasing bid price. This means the bid with the highest price offered will be first in the list and so on. Alternatively, the bids can be sorted after they are retrieved. The list of bids can be stored in a computer file, in memory of the processor performing the auction, be inputted by the keyboard or transmitted over a network in real-time by the bidder's software bidding agreementagent at the time of the auction or can be retrieved by any other means when the progressive second price auction technique is performed. For example, if ten bidders have previously bid for premium bandwidth in an auction, these bids (including the quantity desired and price per unit offered) will be stored in the memory of a computer or other apparatus operating the technique. The bids will then be sorted by a conventional sorting technique to place the previous bids in order from highest and lowest. Alternatively, the bids can be stored in decreasing price order in a linked list as they are received. If the bid to be processed is the first bid made in the auction and no other bid have been stored, the bid will become the first bid in the list.

#### 2. Please amend the paragraph beginning at page 7, line 1 as follows

Step 105 retrieves the new bid  $(S_i)$  to be processed which has at least both a quantity component  $(q_i)$  and price component  $(p_i)$ . The quantity component is in predefined units for the auction, for example, one Mbps (Megabit per second) of bandwidth, and the price will preferably be in the form of price per unit. However, the price can also be entered in the form of total price for the quantity bid and the unit price can be easily calculated. Because the technique utilizes a second price auction technique, the new bidder will be encouraged to optimally submit a bid that is approximately the bidder's true valuation for the resource. The bidder optimally bids his true value because of the operation of the second price auction mechanism as explained above. The new bid can be submitted electronically over a network such as the Internet or can be entered manually into a computer performing the technique. For example, a new bidder desiring

premium bandwidth may type in a bid of \$4 per unit for three units of bandwidth into a computer and transmit the bid to the computer operating the technique over a network. A software agent can be programmed to automatically bid according to the user's needs and the market as represented by the other users' bids.

## 3. Please amend the sixteen (16) paragraphs beginning at page 8, line 6 as follows

Step 111 identifies the next highest bid  $(S_j=(Q_j,P_j))$  where  $S_j$  is the  $j^{th}$  bid;  $Q_j$  is the  $j^{th}$ -quantity component of the  $j^{th}$  bid; and  $P_j$  is the  $j^{th}$  price component of the  $j^{th}$  bid. The first time the auction technique performs this step during a particular auction or allocation, the next highest bid identified will be the bid with the highest price included in the bid list. Since the bids have been sorted by decreasing bid price, the first bid in the list should initially be the highest. If the list is implemented as a linked list, a pointer will be pointing to the highest price bid. Notably, the list could be organized in order of decreasing price, or in some other manner. In that case, modifications, which will be apparent to one of ordinary skill in the art, would need to be made to accommodate this organizational approach.

Step 113 checks if the <u>price component of the current identified bid in the list ( $P_i$ )</u> is greater than or equal to the new bid  $S_i$  in the auction. If the current identified bid in the list is greater than or equal to the new bid price, the technique continues with step 115. If the current bid pointed to in the list is less than the new bid price, than the technique continues with step 117.

Step 115 subtracts the quantity component of the current identified bid in the list from the value of the available quantity  $Q_i$ . Since the price of the current identified bid in the list is greater than the new bid, it will be awarded the quantity of resources that it had requested (to the extent the remaining quantity of resource is not yet assigned to higher bidders during this allocation for the new bid). The amount of the resource which has just been allocated to the higher bid in the bid list is subtracted from the available resource variable  $Q_i$ . If the new bid price is below previous bids from the bid list which in aggregate request a total allocation that equals or exceeds the available

allocation of the service provider (Q), the new bid will be allocated none of the resource because the resource has been allocated to the higher bidders. The new bidder will be forced to increase its bid price in a subsequent bid or obtain the resource from another source. If the current identified (j<sup>th</sup>) bid in the list of retrieved bids requests more resource than is currently available as identified in Q<sub>i</sub>, the entity making the bid will be allocated the remaining portion of the resource. In that case, the new bidder will receive some of the resource that was requested, and Oi will be set to 0.

Step 116 checks if the quantity remaining variable Q<sub>i</sub> is equal to zero. If it is equal to zero, then there is no more resource to be allocated to the new bidder S<sub>i</sub> and the technique is completed for the new bidder S<sub>i</sub> with respect to the bid made. The If the auction is still continuing, or if there will be another allocation in the future, the bidder may then increase its bid price and submit a new bid. If Q<sub>i</sub> does not equal zero, then the testing of received bids in the list continues and the technique goes returns to step 111.111 and the next highest previously submitted bid is identified.

Step 117 is reached if the bid price in the new bid  $S_i$  is high enough such that some amount of quantity remains to be allocated at that bid price. For example, if the total allocation was 9 units and the auction computer has previously received bids (quantity, price) of (3, \$7), (3, \$6), (3, \$2), and the current bid  $S_iS_i$  is (3, \$5), the first two bids of \$7 and \$6 would each be allocated 3 units and 3 units would remain to be allocated ( $Q_i = 3$ ). Step 117 then allocates the proper amount of the resource  $\underline{A}$  to the new bidder  $S_iS_i$  by taking the minimum of (1)  $Q_i$  (the amount remaining) and (2)  $q_i$  (the amount of requested). Thus, the new bid  $S_i$  will be assigned the quantity requested to the extent it is available and not already assigned to higher bids previously received. In the example above, the new bidder would be assigned 3 units.

Steps 119 through 123 calculate the price for the new bidder according to the progressive second price auction technique. Step 119 places the new bid in sorted order into the list of bids. For example, the bid could be added to a linked list such that the bid prices are arranged in descending order. The lowest bid which has previously received an allocation of the resource is then identified. If an initial allocation has not yet

been performed, then the lowest bid is identified which would have been allocated some of the resource if the new bid was not present.

Step 121 then calculates the total cost for the new bidder based upon the progressive second price auction technique. The cost to the new bidder is calculated on an exclusion compensation principle: the new bidder (i) pays for his allocation so as to cover the "social opportunity cost" which that is given by the declared willingness to pay (through their bids) of the users which are excluded by i's presence. The operator and supplier of resources at the auction is thus compensated for the maximum lost potential revenue from the new allocation.

Thus in the previous example, if the lowest bid receiving an allocation of resources was \$2 for three units before the new bid, these three units would be reassigned to the new bidder who just bid \$5 for the three units. The new bidder  $S_iS_i$  would then be charged \$2 for each unit allocated. If an additional bidder now entered the bidding with a bid of (3, \$5.50), the stored bidding list would include the following: (3, \$7), (3, \$6), (3, \$5), (3, \$2) with the first three bids being allocated three units each. The new bidder at \$5.50 would be allocated the three units of the \$5 bidder at a cost of \$5 per unit according to the second price auction rule. Thus allocation prices are always based upon the bids of the bidder with lower bids who had previously received an allocation or would have received an allocation but for the new bidder.

Step 121 shows how the cost for a new bidder who is receiving an allocation of the resource is calculated. For each bid with a lower bid price which received an allocation of the resource before the new bid, the bid price is added to the total cost charged to the new bidder until the allocated quantity matches the requested quantity in the bid. This reflects the shift in allocation from the lower bidders to the new bidder with the higher bid. The amount reallocated for each previous bid is responsive to how much quantity was previously allocated or would have been at that bid level. Thus the cost Ci is equal to Ci plus the price of the unit bid times the minimum of (1) the quantity being reallocated and (2) quantity allocated for the amount excluded from the allocation of each previous bid. If an allocation of 2 units at \$3 is reallocated during the

processing of a new bid of 4 units at \$4, 2 units will be reallocated for that new bid from the lower bid. The quantities allocated in the next higher bids (but below the new bid price) will then be used to further calculate the total cost of the new bid reflecting the displaced allocation.

After the reallocation has been performed for a particular bid, the amount to be reallocated for the new bid is reduced by the minimum of (1) the amount to be reallocated  $a_i$  and (2) the amount allocated at the retrieved bid  $Q_i$  ( $a_i=a_i-m_w(j,a_i)$ ). Thus once the allocation amount requested in the bid has been accounted for by one or more lower bids which have been or would have been allocated, no further costs need to be taken into account.

For each bid j having a lower quantity component than bid i, an amount  $a_i$  is determined as the minimum of the quantity bid by bidder j and the quantity remaining to be allocated  $(Q_i)$  after allocating to the bidders having a higher bid price than the new bid i. The cost charged to bidder i is calculated, for each bid as  $C_i = C_i + P_j \times [a_j - MIN (a_j, Q_{j-a_j})]$ . The amount to be allocated  $a_i$  is then decremented according to  $a_j = a_j - MIN (a_j, a_j)$ . In step 122,  $Q_i$  is decremented according to:  $Q_i = MAX(Q_i - q_i, 0)$ .

Step 123 then identifies the next higher  $P_j$  to be used in calculating  $C_j$  lower  $P_j$ . The process then continues with step 125. Step 125 checks if  $a_i$  (the amount remaining be reallocated) is zero or if a  $P_j$  exists which is greater than  $P_i$  no lower bids exist. If one of these conditions is true, then the process is complete for the new bid. If there are additional lower  $P_j$  to process and there is some resource which is to be reallocated, the process  $P_j$ -continues with step 121.

Figures 2 and 3 are graphical examples of the change in an allocation of resources in response to a new bid being processed and illustrate the technique for determining the cost to be charged to a new bidder. Figure 2 depicts and prior allocation of resources based upon previous bids before the new bidder has entered the auction a new bid is processed. In this example, the total quantity of resources (Q<sub>T</sub>) is thirty-five units as indicated in box 203. In this example, the following bids are retrieved as

indicated in box 205: the first retrieved bid  $S_1$  is for 7 units at \$1 per unit; the second retrieved bid  $S_2$  is for 18 units at \$2 per unit; the third retrieved bid  $S_3$  is for 5 units at \$3 per unit and the fourth retrieved bid  $S_4$  is for 10 units at \$10 per unit. Note that in this example, the bids are listed by increasing price, rather than decreasing price as previously described.

Graph 201 shows a horizontal axis indicating bid quantity 207 and a vertical axis indicating bid price 209. The bids located to the right of the vertical axis are allocated a portion of the resource being auctioned. The bids are initially allocated according to the highest bid price. Thus the fourth bid  $S_4$  (the highest bid price) is indicated by bar 211. Bar 211 shows that the entire 10 units requested by the bid have been allocated. Bar 213 shows the allocation for the third bid  $S_3$  which received all of its five requested units. Bar 215 shows the allocation for the second bid  $S_2$  which received all of its 18 requested units. After the bids  $S_4$ ,  $S_3$  and  $S_2$  were allocated, only two units remained unallocated (35-10-5-18 = 2). Thus, bar 217 indicates two units being allocated to bid  $S_1$  and five units requested by  $S_1$  not receiving an allocation (shown to the left of the vertical axis). The price levels on the graph indicate the bid price and the not price actually paid. The cost of the successful bid is calculated using the progressive second price auction technique described in connection with Figure 1-cost actually charged to the bidder, which is calculated in accordance with the principals described herein.

Figure 3 shows a graphical example of how the allocation changes if a new fifth bid comes into the auction as  $S_5$  (5, \$2.50) according to the present technique. Figure 3 shows the graph 201 from Fig. 2, but now with the new allocation due to after the processing of the new bid  $S_5$ . Graph 301 has a horizontal axis indicating bid quantity 307 and a vertical access indicating the bid price 309. The total level of quantity available  $Q_T$  is still 35 units as indicated in box 203. The new bid  $S_5$  is shown in the bid list 305 and is placed in the order of prices bid. Bid  $S_5$  with a bid price of \$2.50 is shown between  $S_2$  and  $S_3$ . The graph now reflects the allocation to each bid after the new bid is taken into account.

In Figure 3,After the processing of bid  $S_5$ , the entity withmaking bid  $S_4$  is againstill allocated 10 units as shown by bar 311. The person withentity making bid  $S_3$  is again allocated 5 units as shown by bar 313. The next highest bid is  $S_5$  so the bidder of  $S_5$  is allocated the five units requested. as shown by bay 319. Between  $S_4$ ,  $S_3$  and  $S_5$ , twenty units have been allocated thus only 15 units remain (35-10-5 = 15). The maker of the next highest bid,  $S_2$ , has requested an allocation of 18 units. However, since only 15 units remain to be allocated, so  $S_2$  receives only 15 barsunits as shown by bar 315. This is three less than iswas previously allocated to this bidder in the example shown in Fig. 2. Bid The entity making bid number one, who requested seven units, is now allocated zero units as shown by bar  $\frac{317317}{3}$ , which is to the left of the vertical axis.

The cost of the new allocation to new bidder  $S_5$  is the amount displaced 321-by the reallocation-, shown in crosshatch 321 in Fig. 2. Thus the total cost for the five units reallocated and displaced by the bid  $S_5$  is (3 units x \$2) + (2 units x \$1) = \$8. The unit cost for the fifth bidder  $S_5$  is the total cost divided by the number of units allocated or 8/5 = \$1.6. Thus bidder five will pay \$1.6 per unit even though he bid \$2.5 per unit according to the progressive second price auction technique.

### 4. Please amend the paragraph beginning at page 13, line 22 as follows

Figure 5 shows an allocation graph similar to the graphs shown in Figures 2 and 3. Box 501 shows the valuator function 10q (or ten times the quantity) where the quantity is shown on the horizontal axis and utility is shown in the vertical axis. In the graph 503, the bids discussed in the example of figure Figure 4 are shown. The horizontal axis indicates quantity allocated 505 and the vertical axis is the price of the bid 507. The total quantity to be allocated is one hundred units. The bids for the auction are indicated in box 509. The allocation to bid  $S_1$  is indicated by bar 511519; the allocation to bid  $S_2$  is indicated by bar 513517; the allocation to bid  $S_3$  is indicated by bar 519511. Bid  $S_{51}$  is only allocated twenty units of all of the overall units bid. In order to determine the optimal bid by a new playplayer  $S_6$  with having a valuation function shown in box 501, the derivative of the valuation function is taken to determine the slope of the demand.

The derivative of the new bidder's valuation function is shown in graph 503 as line 521. Every quantity indicated in a bar below line 521 is more valuable to the new bidder than those quantities bid by other bidders with lower price components. Thus the new bidders bidder would want to bid \$10 in order to obtain seventy of the total units which includes the twenty units of line 513, the twenty units of line 515, ten units of line 517 and twenty units of line 519. Additional units could cost above the calculation for the unit so a higher bid would not be efficient. The actual cost which the new bidder will pay is calculated using the technique described in connection with Figure 1.previously. The thirty units shown by bar 511 were bid at a price of \$12 which is above the price of \$10, the utility derived by the new bidder. Thus the new bidder would obtain 70 units at a bid price of \$10. While a straight line derivative function is shown in the example, more complicated valuation functions can be used to model the correct bidding strategy in a similar manner.

### 5. Please amend the paragraph beginning at page 21, line 19 as follows

Figure 10 is a diagram of an Internet Service Provider computer. The ISP network bandwidth is controlled by a computer which preferablemay includes a central processing unit 1001, a memory 1003 connected to the CPU 1001, an input/output (I/O) port 1005 and a databus 1007 connecting processor 1001, memory 1003 and I/O port 1005. In the example of allocating premium bandwidth, the ISP preferably operates computer can operate the progressive second price auction technique. Program instructions implementing the progressive second price auction technique are stored in memory 1003 which are executed by processor 1001 to perform the technique as described in connection with Figure 1. 'As the ISP receives bids for an auction, the bids can be stored in memory 1003. Alternatively, the bids can be stored in a memory outside of the ISP but which is accessible to the ISP. I/O port 1005 is connected to one or more networks which that the Internet Service Provider manages.